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Laminar natural convection from constant heat flux helical coiled tubes

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Abstract—An experimental investigation of a steady state natural convection from uniformly heated helical coiled tubes oriented horizontally in air has been made. Average heat transfer coefficients are obtained for laminar natural convection. The experiments have been carried out for four coils and for various values of heat fluxes of 500–5000 W m^{-2}. The data are correlated with Rayleigh number using the coil tube diameter as the characteristic length. It is obtained that average heat transfer coefficient decreases with increasing the number of coil turns up to (or close to) the middle turn then a transition to turbulent begins. Correlation for all heat fluxes for all coils using the horizontal coil axis distance as a characteristics length is also presented. © 1998 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

Helically coiled tubes are used in many industrial applications, such as in refrigerating and HVAC applications. They are also used in steam generator and condenser designs in power plants for their large surface area per unit volume. In spite of their widespread use, the only information available on natural convection from such coils is that studied experimentally by Ali [1]. He has developed correlations for turbulent natural convection from vertical helical coils in water. In this experiment 10 coils were used with four coil diameter to tube diameter ratios and five pitch to tube diameter ratios, the tube diameter used was 0.008 or 0.012 m. In preparation of this manuscript I had the chance to look at the paper by Xin and Ebadian [2] who reported an experimental study on natural convection heat transfer from helicoidal pipes in air in vertical and horizontal orientations. In their experiment three test coils were tested, the tube diameter of two of them is 0.0127 m and the third is 0.0254 m. For the coils oriented horizontally, the study was concentrated on the middle turn of each coil only and they conclude that the peripheral average Nusselt number distribution around the middle turn is almost periodic. Their correlation for the three middle turns of the three test coils is given by

$$Nu_{\rm d} = 0.318 Ra_{\rm d}^{0.293}, \quad 5\times10^{3} \leqslant Ra_{\rm d} \leqslant 1\times10^{5}. \tag{1}$$

It should be noted that this correlation is valid only at the middle turn of the coils which neglect all the other turn effects and also the end effects on the average heat transfer coefficient. Therefore, it cannot apply to the coil as a unit to predict the heat transfer coefficient from such horizontal coils.

On the other hand, analytical, numerical, and experimental studies of natural convection heat transfer from vertical and horizontal plates and cylinders are available in the literature. Many experiments have been done to study the heat transfer coefficient inside the coils since the criterion for transition from laminar to turbulent in curved pipes was established by Ito [3]. Such experiments are those by Rogers and Mayhew [4], Seban and Mclaughlin [5], and Garimella *et al.* [6].

Laminar and turbulent free convection to air from flat plates and cylinders are reported by many authors for different ranges of *Ra*. Churchill and Chu [7] have developed an expression for the mean *Nu* over a horizontal cylinder for all *Ra* and *Pr* in terms of the model of Churchill and Usagi [8]. A similar correlation was developed by Churchill and Chu [9] for natural convection from a vertical plate. Laminar natural convection from a uniformly heated surface of an inclined cylinder to air was studied experimentally by Arabi and Salman [10]. Their correlation for uniform heat flux of 1000 W m^{-2} and for horizontal cylinder can be presented by

$$Nu_x = 0.158 Ra_x^{1/3}. \tag{2}$$

Their correlation for a vertical cylinder using the total cylinder length is 2% higher than those given by McAdams [11].

This paper presents the results of an experimental investigation of laminar natural convection heat transfer from uniformly heated horizontal coils in air. This study focuses on the determination of average heat transfer coefficients including all the turns of each coil for constant heat flux. Nusselt numbers have correlated with Rayleigh numbers for various values of heat fluxes using the coil tube diameter or horizontal coil axis distance as characteristic lengths.

NOMENCLATURE

A	surface area [m^2]
d	tube diameter [m]
D	coil diameter [m]
g	acceleration due to gravity [m s^{-2}]
h	heat transfer coefficient [W m^{-2} K^{-1}]
H	coil horizontal height [m]
k	thermal conductivity [W m^{-1} K^{-1}]
L	coil length [m]
N	number of coil turns
Nu	Nusselt number, hd/k, or hx/k
P	coil pitch [m]
Pr	Prandtl number, ν/α
q''	heat flux [W m^{-2}]
Ra	Rayleigh number, $g\beta(t_s-t_a)d^3/\nu\alpha$, or $g\beta x^3(t_s-t_a)/\alpha\nu$
t	temperature [°C]
t_s	average coil surface temperature at each turn
x	coil horizontal axis.

Greek symbols

α	thermal diffusivity [m^2 s^{-1}]
β	coefficient of thermal expansion [K^{-1}]
θ	bulk arithmetic mean temperature [°C]
ν	kinematic viscosity [m^2 s^{-1}].

Subscripts

a	ambient condition
d	tube diameter (characteristic length)
i	inner
o	outer
s	slanted.

Section 3 describes the experimental set-up and procedure. This is followed by the analysis of experiments in Section 4 with results and discussion in Section 5. Conclusions are given in Section 6.

2. EXPERIMENT SET-UP AND PROCEDURE

A schematic diagram showing the geometry of the coils used is sketched in Fig. 1(a). The coils were formed from initially straight stainless steel heating tube 'electric stove oven replacement element', 0.0066 m in outside diameter with wall thickness of 0.0005 m. Only a very slight ellipticity of the cross section and distortion of wall thickness are introduced by the bending process. The outer diameter and the wall thickness of every coil was measured by using a sample cut of that coil. Table 1 lists the dimensions and parameters of the coils used in this investigation. To allow for the obliquity of the helix the slanted outer turn diameter D_s (Fig. 1(a)) for each turn is measured using a vernier caliper and the helix coil diameter is obtained from the following equation

$$D = \frac{\Sigma\sqrt{D_s^2 - \left(\frac{P}{2}\right)^2}}{N}.$$

The effective coil length was measured before forming and the coil turns were separated from each other using steel sheet spacers with a specific length to fix the pitch of the coil. The coil is oriented horizontally using two vertical bars in a room away from the openings of the ventilation and air conditioning system to minimize any possible forced convection. Temperatures are measured with calibrated iron constantan (type J) thermocouples at the outer surface along each turn at three locations 120° intervals on the circumference (Fig. 1(b)). Those thermocouples are connected to a multichannel data recorder

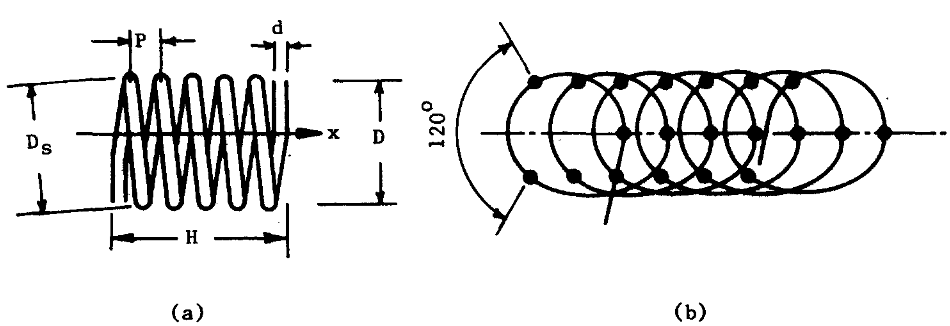

Fig. 1. Schematic of coils (a) physical parameters (b) thermocouple locations.

Table 1. Physical dimensions of the test coils

Coil no.	d_i (m)	d_o (m)	D (m)	L (m)	N	D/d_o	H/d_o	P (m)	A_s (m^2)
1	0.0056	0.0066	0.158	1.885	4	23.94	10.23	0.015	0.0391
2	0.0056	0.0066	0.158	2.365	5	23.94	15.70	0.019	0.0490
3	0.0056	0.0066	0.109	1.885	6	16.45	10.23	0.010	0.0391
4	0.0056	0.0066	0.109	2.513	8	16.45	15.70	0.012	0.0521

(PM8237A) to record their signals when they reach steady state. Heating of the coil was accomplished by alternating current which entered the coil through a variable a.c. supply. The power consumed by the coil was measured by a Wattmeter and the heat flux per unit surface area of the coil was calculated by dividing the consumed power to the coil surface area and it assumed to increase from 500–5000 W m^{-2}. For each coil, current and volt was controlled such that the required heat flux per unit area was closer to the assumed value. The procedure outlined above is used to generate natural convection heat transfer data in air (Prandtl number ≈ 0.71).

3. ANALYSIS OF EXPERIMENT

The average heat transfer coefficient h at each coil turn is calculated by

$$h = \frac{q''}{t_s - t_a}, \quad t_s = \frac{t_{s1} + t_{s2} + t_{s3}}{3}$$

where t_{s1}, t_{s2}, and t_{s3} are the circumferential measured surface temperatures along each coil turn 120° apart and t_s is the average temperature along each turn, t_a is the air ambient temperature, and q'' is the electrical power input, assumed to be dissipated uniformly per unit surface area of the coil. The surface area was calculated from $A = \pi dL$, and the coil height in horizontal direction is defined as $H = NP + d$ (Fig. 1(a)). Direct temperature measurements of the ambient air temperature were made before the run and physical properties were calculated (Bejan [12]) using the bulk arithmetic mean temperature

$$\theta = 0.5[t_s + t_a].$$

The uncertainty analysis in calculating the heat transfer coefficient was made using equation (3.2) in Holman [13] and it was found to be 2.6% at most. Average Rayleigh number Ra_d and Nusselt number Nu_d were generated using tube diameter as the characteristic length to determine the range of Rayleigh number and Nusselt number and its relation to the region of natural convection.

4. RESULTS AND DISCUSSION

Experimental data were obtained with the coils oriented in horizontal position in air over the heat flux range $500 \leqslant q'' \leqslant 5000$ W m^{-2} and the bulk arithmetic temperature range $28 \leqslant \theta \leqslant 110$°C. Within these parameter ranges the maximum average Rayleigh number is approximately 1.8×10^3; and the maximum Nusselt number is about 10 when d_o is used as a characteristic length. The average Nusselt number distribution for coil (1) is shown in Fig. 2 vs. the number of coil turns where the heat transfer coefficient drops down up to the third turn, then becomes higher on the fourth turn. The reason we believe for this could be because the thermal boundary layer becomes thicker on the second and third turns due to the convergence of the plume towards the middle of the coil leaving the fourth turn subjected to the end effects and hence increasing the heat transfer coefficient. Therefore, up to the third turn the flow can be classified as laminar and beyond that as transition to turbulent and as the heat flux increases the Nusselt number variation become more significant. Figure 3 shows the same behavior for some samples of heat fluxes for coil (2) where the laminar region reduced to the second turn while transition develops after that. Furthermore, Fig. 4 shows the variation of Nusselt number of coil (3) for some selected values of heat flux which indicates that the laminar region up to turn number three and after that transition develops as explained earlier. In Fig. 5 the variation of Nusselt number for coil (4) is presented where the laminar region extended to turn number four followed by a transition region. The overall average Nusselt numbers for all turns of the four test coils for $q'' = 500$, and 1000 W m^{-2} are shown vs. Rayleigh number in Fig. 6 on a logarithmic scale. Least-squares power law fit through the data set yields the following correlations for 500 and 1000 W m^{-2}, respectively,

$$Nu_d = 10\,824.2 Ra_d^{-1.196}, \quad 340 \leqslant Ra_d \leqslant 645 \quad (3)$$

$$Nu_d = 187\,508 Ra_d^{-1.526}, \quad 728 \leqslant Ra_d \leqslant 938. \quad (4)$$

The maximum deviation between the experimental data and the correlations are 1.3 and 1.6%, respectively. Figure 7 shows the variation of Nusselt number against Rayleigh number on a log scale for some

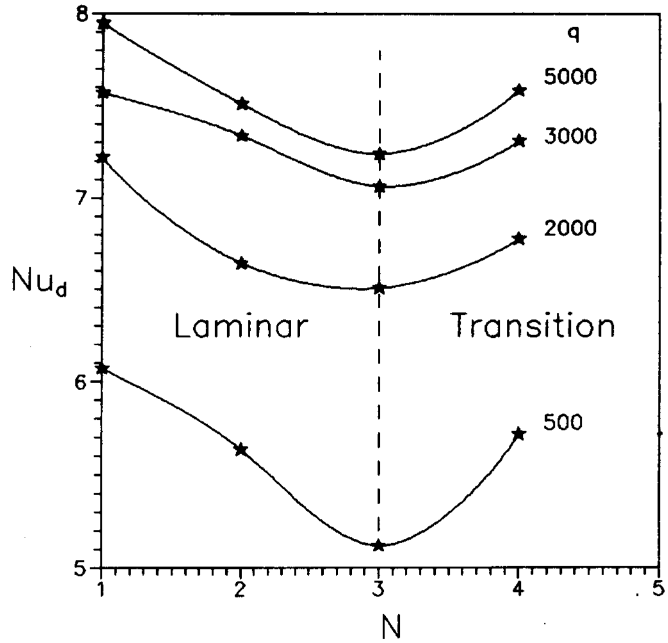

Fig. 2. Average Nusselt number along the coil (1) for various values of heat flux showing laminar and transition regions.

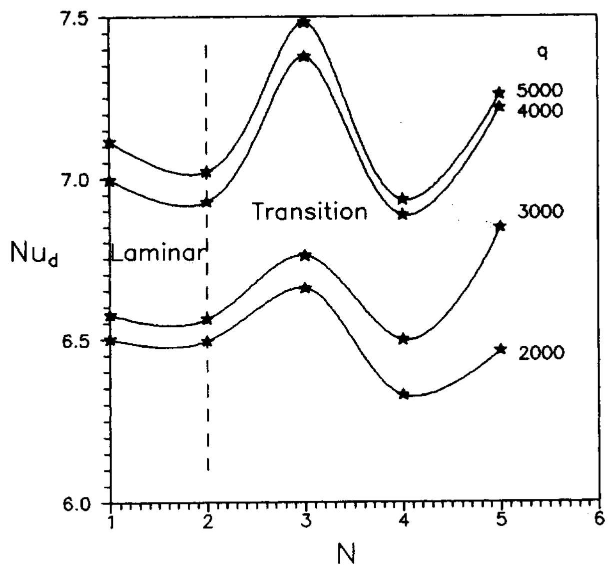

Fig. 3. Average Nusselt number along the coil (2) for various values of heat flux showing laminar and transition regions.

selected values of heat flux and correlations of the form

$$Nu_d = a\,Ra_d^b \quad (5)$$

are obtained. The coefficients a, b and Ra range, the maximum deviation between the data points and the correlation, and the corresponding heat flux are given in Table 2. Since there is no data available in the literature on this subject with which to compare, the coil horizontal axis distance x corresponding to each coil turn was used as a characteristic length in the nondimensional group to compare the present results with those of [10] for a horizontal cylinder in air for $q'' = 1000\ \mathrm{W\ m^{-2}}$. Equation (2) was plotted as dashed lines in Fig. 8 for comparison with the present results

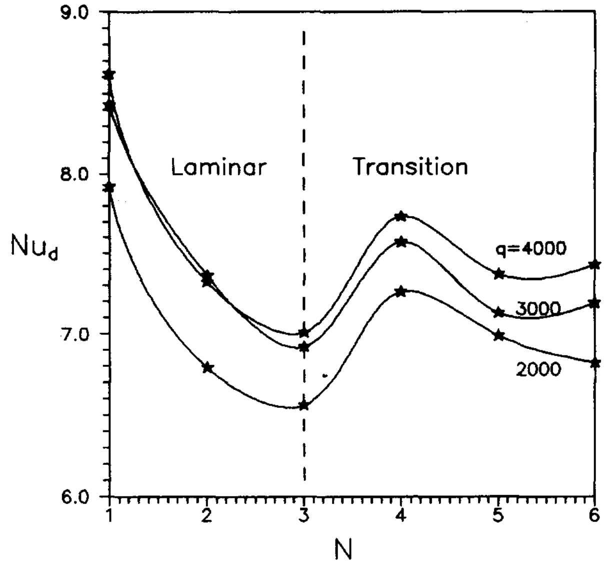

Fig. 4. Average Nusselt number along the coil (3) for various values of heat flux showing laminar and transition regions.

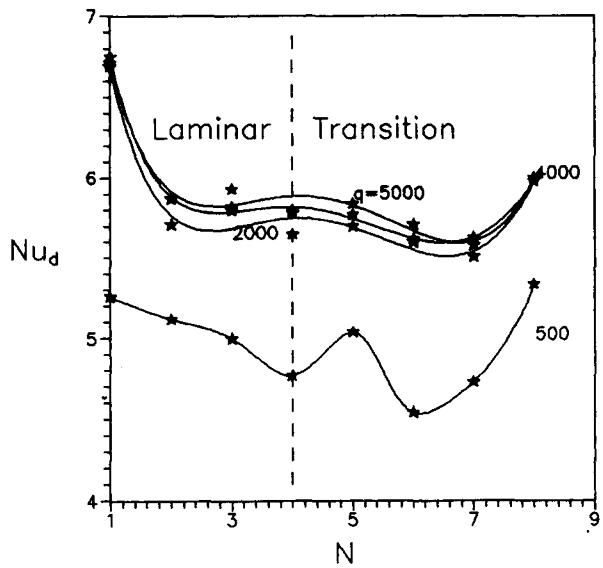

Fig. 5. Average Nusselt number along the coil (4) for various values of heat flux showing laminar and transition regions.

for the same heat flux of 1000 W m^{-2} for the four coils used. A least-square power law fit through the data set yields the following correlation

$$Nu_x = 1.0338 Ra_x^{0.299} \tag{6}$$

and it should be noted, that other fluxes give similar correlations. This comparison between correlations (6) and (2) show that using coils enhances the heat transfer coefficient over that of a horizontal cylinder. Figure 9 was constructed to obtain the following overall correlation for all heat fluxes $1000 \leqslant q'' \leqslant 5000$ W m^{-2} for all coils used in this experiment

$$Nu_x = 0.9125 Ra_x^{0.301}, \quad 2.5 \times 10^3 \leqslant Ra_x \leqslant 5 \times 10^6. \tag{7}$$

Although the exponent in equations (6) and (7) is sensitive to experimental scatter in the data, the results are clearly adequate to show that the exponent is less than 1/3. The maximum deviation between the

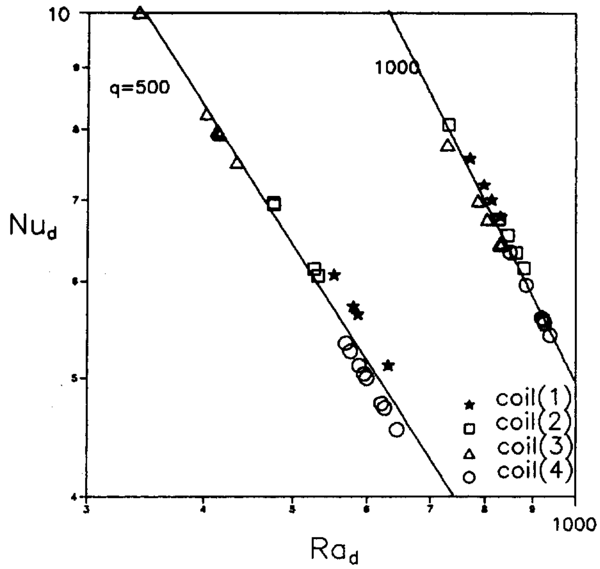

Fig. 6. Average Nusselt number against Rayleigh number for all the coils used for $q'' = 500$ and 1000 W m^{-2}.

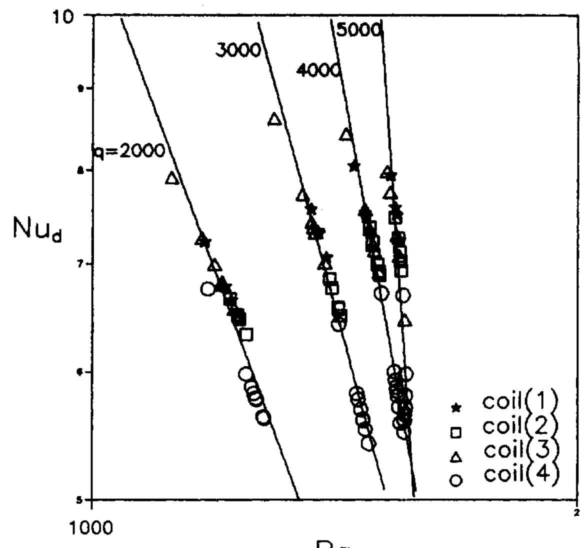

Fig. 7. Average Nusselt number against Rayleigh number for all the coils used for various values of heat flux.

Table 2. Coefficients a, b, Ra range, and the maximum deviation between the experimental data and the correlations for equation (5) for various values of heat flux

q''	a	b	Ra	Maximum deviation (%)
2000	1.198E9	−2.678	1119 : 1277	3.8
3000	1.832E12	−3.632	1296 : 1486	0.9
4000	1.253E18	−5.431	1437 : 1554	1.0
5000	4.988E45	−14.061	1531 : 1567	1.3

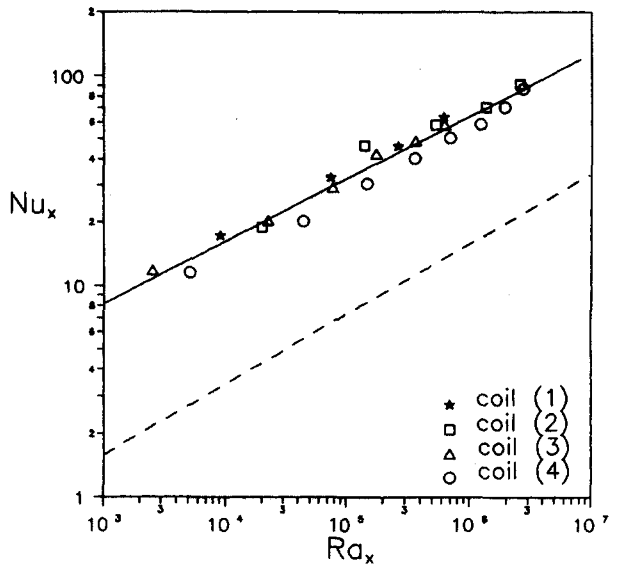

Fig. 8. Comparison of correlating equation (solid line) for horizontal coils with correlation (2) for horizontal cylinder (dashed line) for the same $q'' = 1000\ \mathrm{W\ m^{-2}}$.

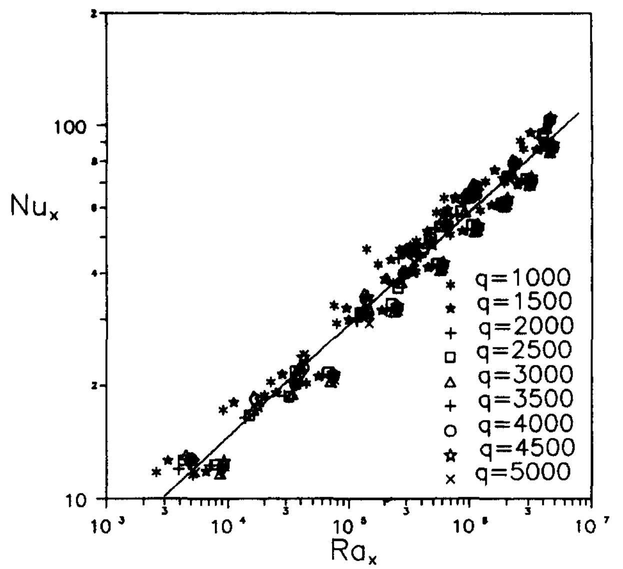

Fig. 9. Overall average Nusselt number against Rayleigh number for all coils used for various values of heat flux.

experimental data and the correlations (6) and (7) following the expression in [13] is 2%. Furthermore, it is clear that the average heat transfer coefficient at any x station along the horizontal coil axis distance decreases slightly with x for equations (6) and (7), respectively, as

$$h_x \propto x^{-0.104}, \quad h_x \propto x^{-0.097}. \tag{8}$$

Equation (8) indicates that we have a laminar regime since the turbulent regime is characterized by a constant h similar to natural convection from flat plate (Burmeister [14]) and from horizontal and vertical cylinders in [10] and [15], respectively.

5. CONCLUSIONS

It is shown experimentally, that the correlation covering the natural convection regime of the coils in horizontal orientation in air for all heat fluxes is given in the form

$$Nu_d = a\,Ra_d^b$$

where a and b are the constants given in Table 2 corresponding to each heat flux in laminar regime. In all the coils used it is obtained that laminar regime starts first followed by transition to turbulent regime almost at the middle of the coil. This laminar regime is characterized by a decrease in the heat transfer coefficient while transition to turbulent regime is characterized by a wave variation of average heat transfer coefficient with increasing the number of coil turns as shown for coil (2) and (3) in Figs. 3 and 4, respectively.

Using the horizontal coil axis distance x corresponding to each turn as a characteristic length shows that the overall correlation for all heat fluxes and for all coils used is given by

$$Nu_x = 0.9125 Ra_x^{0.301}$$

which results in a decrease in heat transfer coefficient with the horizontal coil axis distance indicating a laminar regime in the whole x range.

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